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CONCERNING FIRST COUNTABLE SPACES. III

BY

G. M. REED

ABSTRACT. The primary purpose of this paper is (1) to provide a "real" example of a regular first countable T_1-space which has no dense developable subspace and (2) to provide a new technique for producing Moore spaces which fail to have dense metrizable subspaces. Related results are established which produce new examples of noncompletable Moore spaces and which show that each regular hereditary M-space with a G_δ-diagonal has a dense metrizable subspace.

The existence of dense metrizable subspaces in a given space S has been shown useful (1) to determine the equivalence of chain conditions and separability ([4], [14], and [16]), (2) to determine if S satisfies the Baire property ([1], [18], and [24]), (3) to determine whether S is densely embeddable in a space of the same type satisfying various completeness conditions ([1], [4], [16], and [18]), and (4) to determine if there exists a space X the same type as S in which each open set contains a copy of S [23]. The inspiration for this work has been the considerable work done in [25], [4], [5], [11], [15], [16], [17], and [18] concerning the existence of dense metrizable subspaces in Moore spaces, i.e., regular developable spaces.

In [13] and [14], the author has investigated conditions under which first countable spaces (all spaces are to be T_1) have dense developable and dense metrizable subspaces. Surprisingly, the major problem in this investigation has been producing first countable spaces which fail to have such subspaces. The only known examples of such spaces are all nonseparable spaces which satisfy the countable chain condition. In [14], it was shown that hereditarily Lindelöf, nonseparable spaces have no dense developable subspaces. In [4], it was shown that nonseparable Moore spaces with the countable chain condition have no dense metrizable subspaces. These are the only techniques presently known for producing the required examples. Thus, other than the consistency of Souslin spaces, there is no known example of a regular first countable space which has no dense developable subspace. And there are very few known examples ([21], [22], and [10]) of Moore spaces which have no dense metrizable subspaces. It is the primary

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purpose of this paper to provide a "real" example of a regular first countable space which has no dense developable subspace and to provide a new technique for producing Moore spaces which fail to have dense metrizable subspaces.

In Theorem 1, the author shows that a regular (in fact, hereditarily paracompact) first countable space due to Miščenko in [6] and Aull in [2] has no dense developable subspace. In [19] and [20] the author has developed a technique which associates a Moore space to each regular first countable space. In this paper it is shown that if a given regular first countable space has no dense developable subspace, then the associated Moore space has no dense metrizable subspace. Thus, by using Theorem 1, the author obtains a new example of a Moore space which fails to have a dense metrizable subspace. Furthermore neither the space of Theorem 1 nor its associated Moore space has the countable chain condition. Related results are established which (1) produce new examples of noncompletable Moore spaces and (2) provide a partial answer to a question raised in [14] by showing that each regular hereditary M-space with a G_δ-diagonal has a dense metrizable subspace.

PRELIMINARIES. A development for a space S is a sequence $G_1, G_2, \ldots$ of open coverings of S such that for each $p \in S$ and each open set D containing p, there exists an n such that each element of G_n containing p is contained in D. A regular developable space is a Moore space. A Moore space is complete provided it has a complete development, i.e., a development $G_1, G_2, \ldots$ such that if $M_1, M_2, \ldots$ is a sequence of closed sets such that for each i, M_i is contained in an element of G_i and contains M_{i+1}, then $\bigcap M_i \neq \varnothing$. A Moore space is completable provided it can be embedded in a complete Moore space. A space S is screenable provided that for each open covering G of S there is an open covering $H = \bigcup_{i=1}^{\infty} H_i$ of S which refines G such that, for each i, H_i is a collection of mutually exclusive open sets. In [3], it was shown that a Moore space is metrizable if and only if it is normal and screenable.

THEOREM 1. *There exists a hereditarily paracompact T_2-space X with a point countable base which has no dense Moore subspace.*

PROOF. Define (X, T) as follows: Denote by A the set of all ordinal numbers which precede the first uncountable ordinal. For each $a \in A$, let $R(a)$ denote the set of all ordinal numbers which precede a and let X_a denote the set of all mappings x of the set $R(a)$ into N, the set of all natural numbers. Now, let $X = \bigcup\{X_a | a \in A\}$. For each $a \in A$ and $x \in X_a$, call a the length of x. Furthermore, say that the element x of X_a is an extension of the element y of X_b if $b < a$ and for $c < b$, $y(c) = x(c)$. Finally, for each $a \in A$, $x \in X_a$, and $n \in N$ denote by $u_n(x)$ the set consisting of the point x and of all $y \in X$ such that y is an extension of x and $y(a) \geqslant n$. It follows that $B = \{u_n(x) | x \in X$ and $n \in N\}$ is a base for the topology T on X. To see this, note that if $y \neq x$ and $y \in u_k(x)$, then $u_n(y) \subset u_k(x)$ for each $n \in N$. Aull showed in [2] that X is a hereditarily paracompact T_2-space with a point countable base.

Now, suppose that there exists a dense subspace K of X such that K is a Moore space. Note that (1) if $x \in X$, then there exists $y \in K$ such that y is an extension of x; and (2) if $x \in K$ and D is an open set in K containing x, then there exists $y \in D$ such that y is an extension of x. Denote by $G_1, G_2, \ldots$ a development for K such that if $g \in G_i$ for some i, then there exist $x \in X$ and $n \in N$ such that $g \subset u_n(x)$. Observe that (3) if $\{x, y\} \subset g \in G_i$ for some i and y is an extension of x, then each $z \in K$ such that z is an extension of y is contained in g. Construct the sequences $x_1, x_2, \ldots$ and $g(x_1), g(x_2), \ldots$ as follows: Let $x_1 \in K$ and let $g(x_1) \in G_1$ such that $x_1 \in g(x_1)$. Let $x_2 \in g(x_1)$ such that x_2 is an extension of x_1 and let $g(x_2) \in G_2$ such that $x_2 \in g(x_2)$ and $g(x_2) \subset g(x_1)$. Continue this process such that for each $i > 2$, $x_i \in g(x_{i-1})$, x_i is an extension of x_{i-1}, $x_i \in g(x_i) \in G_i$, and $g(x_i) \subset g(x_{i-1})$. Then, for each i, let $a_i \in A$ such that $x_i \in X_{a_i}$ and let $a \in A$ such that for each i, $a_i < a$. Denote by x an element of X_a such that for each i, x is an extension of x_i. By (1) and (3) above, there exists $y \in K$ such that y is an extension of x and $y \in g(x_i)$ for each i. Thus, for each i, $\{x_i, y\} \subset g(x_i) \in G_i$ but y is not a limit point of $\{x_1, x_2, \ldots\}$. This contradicts the assumption that K is a Moore space.

THEOREM 2. *Suppose that X is a first countable space which is the union of countably many subsets X_i such that for each i, there exists a collection U_i of mutually exclusive open sets in X covering X_i such that each element of U_i contains only one point of X_i. Then X has a dense screenable, developable subspace.*

PROOF. Let $K_1 = X_1$ and for each $i > 1$, let $K_i = X_i - (\overline{\bigcup_{j=1}^{i-1} X_j})$. It follows that $K = \bigcup_{i=1}^{\infty} K_i$ is dense in X. Consider K as a subspace of X and for each i, let $U_i' = \{u \cap K | u \in U_i$ and $u \cap K_i \neq \varnothing\}$. For each i and each point $x \in K_i$, denote by $g_1(x), g_2(x), \ldots$ a nonincreasing sequence of open sets in X which forms a local base at x and is such that $g_1(x)$ is contained in the element of U_i' which contains x. Finally, for each j and each $i > 1$, let $K_{ij} = \{x \in K_i | x \notin g_j(y)$ for $y \in \bigcup_{n=1}^{i-1} K_n\}$. By the construction of K, it follows that $K = K_1 \cup \{\bigcup_{i=2}^{\infty} \bigcup_{j=1}^{\infty} K_{ij})$. Furthermore K_1 and each K_{ij} are discrete subsets of K. Hence K is the union of countably many discrete subsets and by [13, Lemmas 1.2 and 1.4] is developable. That K is screenable follows immediately.

The following construction was developed by the author in [19] and [20].

CONSTRUCTION OF S_0. Let X_0 be a regular first countable T_1-space. For each $x \in X_0$, denote by $u_1(x), u_2(x), \ldots$ a sequence of open sets in X_0 which forms a local base at x such that for each i, $\overline{u_{i+1}(x)} \subset u_i(x)$. Now for each positive integer m, let $A_m = \{(n_1, n_2, \ldots, n_m) | n_1 = 1$ and for $1 \leqslant i \leqslant m$, n_i is a positive integer$\}$. Let $A = \bigcup_{m=1}^{\infty} A_m$. For each $a = (n_1, n_2, \ldots, n_m) \in A$, denote by S_a a unique copy of X_0. And for each $x \in X_0$, denote by $x_a = (x_{n_1}, x_{n_2}, \ldots, x_{n_m})$ the element of S_a which is identified with x. Let $S_0 = \bigcup\{S_a | a \in A\}$ and define a development for S_0 as follows: For each positive integer j, $a = (n_1, n_2, \ldots, n_m) \in A$, and $p = (y_{n_1}, y_{n_2}, \ldots, y_{n_m}) \in S_a$, let

$$g_j(p) = \{p\} \cup \{(x_{n_1}, x_{n_2}, \ldots, x_{n_m}, x_{k_1}, x_{k_2}, \ldots, x_{k_c}) | \; x \in X_0, c \text{ is a positive integer, for } 1 \leqslant i \leqslant c, k_i \geqslant j, \text{ and } x \in u_{k_1+j}(y) \text{ in } X_0\}.$$

It follows that $G_1, G_2, \ldots$, where for each i, $G_i = \{g_j(p) | p \in S$ and $j \geqslant i\}$, is a development for the Moore space S_0.

Claims. (1) If D is open in S_0, then $D' = \{x \in X_0 | x_a \in D$ for some $a \in A\}$ is open in X_0. Furthermore, if D' is an open set in X_0, then $D = \{x_a \in S_0 | x \in D'$ and $a \in A\}$ is open in S_0.

(2) If K is dense in S_0, then $K' = \{x \in X_0 | x_a \in K$ for some $a \in A\}$ is dense in X_0.

(3) If K' is dense in X_0, then $K = \{x_a \in S_0 | x \in K'$ and $a \in A\}$ is dense in S_0.

(4) For each j and each $a \in A$, $g_j(x_a) \cap g_j(y_a) = \varnothing$ in S_0 if and only if $u_{2j}(x) \cap u_{2j}(y) = \varnothing$ in X_0.

(5) If D' is an open set in X_0 and d is an open set in S_0 such that $\bar{d} \subset D = \{x_a \in S_0 | x \in D'$ and $a \in A\}$, then $d' = \{x \in X_0 | x_a \in d$ for some $a \in A\}$ is open in X_0 and $\bar{d}' \subset D'$.

(6) Suppose $a = (n_1, n_2, \ldots, n_m) \in A$, $M \subset S_a$, and for some i, $D_i = \bigcup\{g_i(y_a) | y_a \in M\}$. Then $D_i = \bigcup_{j=i}^{\infty} D_{ij}$, where for each $j \geqslant i$, $D_{ij} = \{(x_{n_1}, x_{n_2}, \ldots, x_{n_m}, x_{k_1}, x_{k_2}, \ldots, x_{k_c}) | x \in X_0$, $K_1 = j$, c is a positive integer and, for $1 \leqslant n \leqslant c$, $k_n \geqslant i$, $y_a \in M$ and $x \in u_{k_1+i}(y)$ in $X_0\}$. Furthermore, each D_{ij} is open in S_0, $D'_{ij} = \{x \in X_0 | x_b \in D_{ij}$ for some $b \in A\}$ is open in X_0, and if d' is an open set in X_0 such that $\bar{d}' \subset D'_{ij}$ then $d = \{x_b \in D_{ij} | b \in A$ and $x \in d'\}$ is open in S_0 and $\bar{d} \subset D_{ij}$.

In the following theorems, X_0 will denote a regular first countable space and S_0 will denote the Moore space associated to X_0 by the above construction. The terms $u_i(x)$, $g_i(x)$, and G_i will be used as in the construction of S_0.

Theorem 3. *S_0 has a dense screenable subspace if and only if X_0 has a dense screenable Moore subspace.*

PROOF. Suppose that S_0 has a dense screenable subspace. Then by [15, Lemma 2.1 and the proof of Theorem 1.4] there exists a dense screenable subspace K of S such that $K = \bigcup_{i=1}^{\infty} K_i$, where for each i, K_i is discrete in S and there exists a collection V_i of mutually exclusive open sets in S covering K_i such that each element of V_i contains only one point of K_i. For each i and each $a \in A$, let $K_i(a) = K_i \cap S_a$ and denote by $V_i(a)$ a collection of mutually exclusive open sets in S_0 covering $K_i(a)$ such that each element of $V_i(a)$ contains at most one point of $K_i(a)$. Now for each j, let $K_{ij}(a) = \{p \in K_i(a) |$ if $p \in g \in G_j$, then g is contained in the element of $V_i(a)$ which contains $p\}$. Thus $K = \bigcup \{K_{ij}(a)\}$. Finally, for each $K_{ij}(a)$, consider $X_{ij}(a) = \{x \in X_0 | x_a \in K_{ij}(a)\}$. It follows from claim (4) that $H_{ij}(a) = \{u_{2j}(x) | x \in X_{ij}(a)\}$ is a collection of mutually exclusive open sets in X_0 covering $X_{ij}(a)$ such that each element of $H_{ij}(a)$ contains only one point of $X_{ij}(a)$. Hence $X = \bigcup\{X_{ij}(a)\}$ satisfies the hypothesis of Theorem 2 and X has a dense screenable Moore subspace. Furthermore, since K is dense in S_0, X is dense in X_0. Thus, X_0 has a dense screenable Moore subspace.

Suppose that X_0 has a dense screenable Moore subspace. As above let X be such a subspace such that $X = \bigcup_{i=1}^{\infty} X_i$ where for each i, X_i is discrete in X and H_i is a collection of mutually exclusive open sets in X_0 covering X_i such that each element of H_i contains only one element of X_i. For each i and each j, let $X_{ij} = \{x \in X_i | u_j(x)$ is contained in the element of H_i which contains $x\}$. Now for each i, each j, and each $a \in A$, let $K_{ij}(a) = \{x_a \in S | x \in X_{ij}\}$. It follows that $U_{ij}(a) = \{g_j(p) | p \in K_{ij}(a)\}$ is a collection of mutually exclusive open sets in S_0 covering $K_{ij}(a)$ such that each element of $U_{ij}(a)$ contains only one point of $K_{ij}(a)$. Thus, $K = \bigcup\{K_{ij}(a)\}$ is a dense screenable subspace of S_0.

THEOREM 4. *If S_0 has a dense metrizable subspace, then X_0 has a dense screenable Moore subspace.*

PROOF. Each metrizable space is screenable. Thus if S_0 has a dense metrizable subspace, then by Theorem 3, X_0 has a dense screenable Moore subspace.

COROLLARY 5. *If X_0 is the space of Theorem* 1, *then the associated Moore space S_0 has no dense metrizable subspace.*

THEOREM 6. *If X_0 has a dense metrizable subspace, then S_0 has a dense metrizable subspace.*

PROOF. Let $K = \bigcup_{i=1}^{\infty} K_i$ denote a dense metrizable subspace of X_0 such that for each i, K_i is discrete in K. For each $a = (n_1, n_2, \ldots, n_m) \in A$, let $S'_a = \{(x_{n_1}, x_{n_2}, \ldots, x_{n_m}) \in S_a | x \in K\}$. It follows that $S' = \bigcup\{S'_a | a \in A\}$ is the required dense metrizable subspace of S_0. To see this, consider K_i for each i. Since K is metrizable, there exists a discrete collection H_i of mutually exclusive open sets in K covering K_i such that each element of H_i contains only one point of K_i. For each j, denote by K_{ij} the set of all $x \in K_i$ such that $u_j(x)$ is contained in the element of H_i which contains x. Now, for each $a = (n_1, n_2, \ldots, n_m) \in A$ consider $S'_a(i, j) = \{(x_{n_1}, x_{n_2}, \ldots, x_{n_m}) \in S_a | x \in K_{ij}\}$. It follows that $V_a(i, j) = \{g_j(p) \cap S' | p \in S'_a(i, j)\}$ is a discrete collection of mutually exclusive open sets in S' covering $S'_a(i, j)$ such that each element of $V_a(i, j)$ contains only one point of $S'_a(i, j)$. Thus, $S' = \bigcup\{S'_a(i, j) | i \in N, j \in N, a \in A\}$ and by [13, Lemmas 1.3 and 1.4] is metrizable.

REMARK. Is it true that S_0 has a dense metrizable subspace if and only if X_0 has a dense metrizable subspace? Or, more generally, must each screenable Moore space have a dense metrizable subspace? An affirmative answer to the latter question would generalize several results in [5], [11], and [15]. Recently, Tall and Przymusiński in [12] have very significantly shown that is consistent with set theory for there to exist a normal subspace of a nonseparable Moore space with the countable chain condition given in [10] which is also nonseparable and has the countable chain condition. Also, W. G. Fleissner has announced that it is consistent with set theory that each normal Moore space has a dense metrizable subspace. Thus, the proposition that each normal Moore space has a dense metrizable subspace is now known to be independent of set theory.

wd-NORMALITY. In [15], the author defined a space S to be weakly densely normal (wd-normal) provided that if D is an open set in S and H is a closed subset of D, then there exists a sequence $d_1, d_2, \ldots$ of open sets in S such that $H \subset \overline{\bigcup_{i=1}^{\infty} d_i}$ and for each i, $\overline{d_i} \subset D$. A space S is said to be perfectly wd-normal provided that for each open set D in S there exists a sequence $d_1, d_2, \ldots$ of open sets in S such that $D \subset \overline{\bigcup_{i=1}^{\infty} d_i}$ and for each i, $\overline{d_i} \subset D$. It is easily seen that a wd-normal space in which closed sets are G_δ-sets is perfectly wd-normal. Hence, the two properties are equivalent in Moore spaces. In [15], each Moore space with the countable chain condition was shown to be wd-normal.

AXIOM C. A development $G_1, G_2, \ldots$ for a Moore space S is said to satisfy Axiom C at the point p of S provided that for each open set D in S containing p there exists an n such that each element of G_n intersecting an element of G_n containing p is contained in D. It follows from [8] and [25], that if G is a development for the Moore space S, then $C(G)$, the set of all points in S at which G satisfies Axiom C, is, if nonempty, a metrizable G_δ-subset of S. In [13], the author generalized this concept by defining a subset M of the first countable space S to be C-developable in S provided there exists a sequence $G_1, G_2, \ldots$ of open covers of S such that if $p \in M$, then for each open set D in S containing p there exists an n such that each element of G_n intersecting an element of G_n containing p is contained in D.

THEOREM 7 ([15] AND [18]). *In a Moore space S, the following are equivalent*:

(1) *S has a development G such that $C(G)$ is dense in S.*

(2) *S is wd-normal and has a dense screenable subspace.*

(3) *S can be densely embedded in a developable T_2-space which has the Baire property.*

THEOREM 8 [13]. *In a regular first countable space, the following are equivalent*:

(1) *S has a dense subset which is C-developable in S.*

(2) *S is perfectly wd-normal and has a dense screenable Moore subspace.*

THEOREM 9. *S_0 is perfectly wd-normal if and only if X_0 is perfectly wd-normal.*

PROOF. Suppose X_0 is perfectly wd-normal. Let D be an open set in S_0. For each $a = (n_1, n_2, \ldots, n_m) \in A$ and each i, let $D_i(a) = \bigcup\{g_i(x_a) | x_a \in D \cap S_a$ and $g_i(x_a) \subset D\}$. For each $j \geqslant i$, let $D_{ij} = \{(x_{n_1}, x_{n_2}, \ldots, x_{n_m}, x_{k_1}, x_{k_2}, \ldots, x_{k_c}) | x \in X_0$, $k_1 = j$, c is a positive integer and for $1 \leqslant n \leqslant c$, $k_n \geqslant i$, $y_a \in D \cap S_a$, and $x \in u_{k_1+i}(y)$ in $X_0\}$. For each i and $j \geqslant i$, consider $D'_{ij} = \{x \in X_0 | x_b \in D_{ij}$ for some $b \in A\}$. Since X_0 is wd-normal, there exists a sequence $d'_{ij}(1), d'_{ij}(2), \ldots$ of open sets in X_0 such that $D'_{ij} \subset \overline{\bigcup_{m=1}^\infty d'_{ij}(m)}$ and for each m, $\overline{d'_{ij}(m)} \subset D'_{ij}$. But by claim (6), for each m, $d_{ij}(m) = \{x_b \in D_{ij} | b \in A$ and $x \in d'_{ij}(m)\}$ is open in S and $\overline{d_{ij}(m)} \subset D_{ij}$. It is easily seen from claim (3) that $D_{ij} \subset \overline{\bigcup_{m=1}^\infty d_{ij}(m)}$. Hence, since $D_i(a) = \bigcup_{j=i}^\infty D_{ij}$ and $D = \bigcup_{a\in A} \bigcup_{i=1}^\infty D_i(a)$, it follows that S_0 is perfectly wd-normal.

Suppose S_0 is perfectly wd-normal. Let D' be an open set in X_0. Let $D = \{x_a \in S_0 | x \in D'$ and $a \in A\}$. Denote by $d_1, d_2, \ldots$ a sequence of open sets in S_0 such that $D \subset \overline{\bigcup_{i=1}^\infty d_i}$ and for each i, $\overline{d_i} \subset D$. But by claim (5) for each i, $d'_i = \{x \in X_0 | x_a \in d_i$ for some $a \in A\}$ is open in S_0 and $\overline{d'_i} \subset D'$. Furthermore by claim (2), $D' \subset \overline{\bigcup_{i=1}^\infty d'_i}$. Hence, X_0 is perfectly wd-normal.

COROLLARY 10. *S_0 has a dense subset which is C-developable in S_0 if and only if X_0 has a dense subset which is C-developable in X_0.*

COROLLARY 11. *If X_0 is not perfectly wd-normal, then S_0 is not completable. In fact, S_0 cannot be densely embedded in a developable T_2-space having the Baire property.*

EXAMPLES. There are very few known examples of noncompletable Moore spaces. However, the above results make the production of such spaces much easier. For example, it follows immediately that neither the Michael Line nor the space of countable ordinals with the order topology is perfectly wd-normal. Hence, their associated Moore spaces are noncompletable.

M-SPACES. In [14], the author showed that each regular M-space with a G_δ-diagonal in which closed sets are G_δ sets has a dense metrizable subset. The following lemmas were the basis for that result.

LEMMA 12 [9]. *If X is a regular M-space and H is a discrete subset of X, then there exists a collection U of mutually exclusive open sets in X covering H such that each element of U contains only one point of H.*

LEMMA 13 [14]. *If X is a regular M-space with a G_δ-diagonal then there exists a dense subset K of X such that $K = \bigcup_{i=1}^{\infty} K_i$ where for each i, no point of K_i is a limit point of K_i.*

THEOREM 14. *If X is a regular hereditary M-space with a G_δ-diagonal then X has a dense metrizable subspace.*

PROOF. Let $K = \bigcup_{i=1}^{\infty} K_i$ be the dense subset of X from Lemma 13. For each i, consider $M = K - (\overline{K_i} - K_i)$. Note that M is dense in K and K_i is discrete in M. Thus, since X is hereditarily an M-space, by Lemma 12, there exists a collection U' of mutually exclusive open sets in M covering K_i such that each element of U' contains only one point of K_i. And since M is dense in X, there exists such a collection U in X. Hence, by Theorem 2, X has a dense developable subspace Z. But, Z is both a Moore space and an M-space, and is therefore metrizable.

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DEPARTMENT OF MATHEMATICS, OHIO UNIVERSITY, ATHENS, OHIO 45701